SAT Mathematics Review

Geometry

John L. Lehet
jlehet@mathmaverick.com
www.mathmaverick.com

Failing to Prepare is Preparing to Fail!
Geometry and Measurement

**Geometric Notation**

\[BF\quad\text{The line containing point } B \text{ and } F\]

\[\overline{BF} \quad \text{The line segment with endpoints } B \text{ and } F\]

\[BF\quad \text{The length of line segment } BF\]

\[\overrightarrow{BF} \quad \text{The ray starting at } B \text{ and extending infinitely through } F\]

\[\angle ABF \quad \text{The angle formed by } AB \text{ and } BF\]

\[m \angle ABF \quad \text{The measure of angle } ABF\]

\[\triangle ABF \quad \text{The triangle with vertices } A, B \text{ and } F\]

\[ABFG \quad \text{The quadrilateral with vertices } A, B, F \text{ and } G\]

\[\overline{AB} \perp \overline{FG} \quad AB \text{ is perpendicular to } FG\]
Problem 1: A, B and C all lie on the same line \( l \), if C is the midpoint of \( AB \) and \( AB = 12 \), what is AC?

Problem 2: On the line \( l \) above, if \( CD = 4 \), \( EF = 2 \) and \( CF = 10 \), what is the value of DE?
Geometry and Measurement

**Angles in the Plane**

Opposite angles formed by intersecting lines are equal and are called vertical angles

So, $X = Z$ and $W = Y$

Supplementary angles are Straight Angles and are equal to 180 degrees

So, $X + W = 180^\circ$,

$X + Y = 180^\circ$,

$W + Z = 180^\circ$,

$Y + Z = 180^\circ$

**Problem 1:** In the above diagram, if $X$ is equal to 40 degrees, what is the value of $W$? What is the value of $Z$? What is the value of $Y$?

**Problem 2:** In the above diagram, $l$ and $m$ are parallel, name all angles that are equal to angle $d$? Name all angles that are supplementary to angle $b$?

If two parallel lines ($l$ and $m$) are intersected by a third line, the alternate interior angles are equal for example, $e$ and $d$ are alternate interior angles
**Geometry and Measurement**

**Triangles**

**Equilateral Triangle**
- equal sides \((a=b=c)\)
- equal angles \((x=y=z=60^\circ)\)
- Angles measure 60º

**Isosceles Triangle**
- two equal sides \((a=b)\)
- two equal angles \((x=y)\)

**Right Triangle**
- one angle is 90º \((a = 90)\)
- two sides are perpendicular
- \(a^2 + b^2 = c^2\) (Pythagorean Theorem)

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**Problem 1:** If \(ABC\) is an Isosceles Triangle, such that \(\angle ABC = \angle BAC\) and \(m\angle ABC\) is 40º, what is the \(m\angle ACB\)?

**Problem 2:** If \(ABC\) is a Right Triangle, such that \(m\angle ABC\) is 35º, what is the \(m\angle ACB\) if it is not 90º?
Problem 1: If $ABC$ is a Right Triangle, such that $m \angle ABC$ is $45^\circ$ and $AC = 4$, what is the length of the longest side?
Geometry and Measurement

**Congruent Triangles**

triangles that have the same size and shape

\[ \Delta ABC = \Delta DEF \]

\[
\begin{align*}
AB &= DE = a \\
BC &= EF = b \\
AC &= DF = c
\end{align*}
\]

**Problem 1:** If \( \triangle ABC \) and \( \triangle DEF \) are congruent triangles, and \( AB=5 \) and \( BC=15 \), what is \( EF \)?

**Similar Triangles**

triangles that have the same shape
(corresponding angles are equal)

\[ \Delta ABC \text{ and } \Delta DEF \text{ are similar triangles} \]

sides are proportional

**Problem 2:** If \( \triangle ABC \) and \( \triangle DEF \) are similar triangles, and \( AB=5 \), \( BC=7 \) and \( DE=15 \), what is \( EF \)?
Triangle Inequality

The sum of the lengths of any two sides of a triangle is greater than the length of the third side

\[ AC < 8 + 11 \]

Problem 1: In \(ABC\), \(AB = 3\) and \(AC = 7\), can \(BC\) be 4? Can \(BC\) be 12? What are the ranges of values of \(BC\)?

Triangle Perimeter and Area

Perimeter = \(b + a + c\) (sum of the three sides)

Area = \(\frac{1}{2}bh\)

Problem 2: In the above triangle, if \(a=6\), \(b=4\), \(c=7\) and \(h=5\), what is the perimeter? What is the area?
Geometry and Measurement

**Quadrilaterals**

- **Parallelogram**
  - opposite angles are equal
  - opposite sides are equal
  - \[ P = 2a + 2b \]
  - \[ A = bh \]

- **Rectangle**
  - A parallelogram with right angles
  - \[ P = 2w + 2l \]
  - \[ A = lw \]
  - \[ AC = BD = \sqrt{l^2 + w^2} \]

- **Square**
  - A rectangle with four equal sides
  - \[ P = 4s \]
  - \[ A = s^2 \]
  - \[ AC = BD = s\sqrt{2} \]

**Other Polygons**

A Regular Polygon is a polygon with all sides and angles equal

Determine unknown angle and sides using triangles

<table>
<thead>
<tr>
<th>sides</th>
<th>interior angle sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>180°</td>
</tr>
<tr>
<td>4</td>
<td>360°</td>
</tr>
<tr>
<td>5</td>
<td>540°</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>(n)</td>
<td>(180(n-2)°)</td>
</tr>
</tbody>
</table>
Geometry and Measurement

Circles

O = Origin of Circle – the center
OA = OB = Radius of Circle
AC = Diameter of Circle
(twice the radius OA or OB)
AB = Arc

the line segment AD is tangent to the circle at point A.
AD touches the circle at only point A.

The Diameter of a circle is twice the Radius of the circle
\[ d = 2r \]

The Circumference of a circle is the distance around the circle –

it is analogous to perimeter of a polygon
\[ C = \pi d = 2 \pi r \]

The Area of a circle is the amount of space within the circle –
\[ A = \pi r^2 \]

Problem 1: Given a circle with center O and area 16\( \pi \).
Points A and B are on the circle and angle OBA is 30º.
Find the length of line segment AB.
Geometry and Measurement

Solid Figures

Rectangular Solid
Think of a Cardboard Box
$V = lwh$
$SA = 2lw + 2lh + 2wh$

Cube
A Special Rectangular Solid in which $l = w = h = s$
$V = s^3$
$SA = 6s^2$

Cylinder
Think of a can of soup
$V = \pi r^2 h$
$SA = 2\pi r^2 + 2\pi rh$

Rectangular Solid Unfolded
“six rectangles”

Cube Unfolded
“six squares”

“circle on top, circle on bottom, rectangle in the middle”

Cylinder Unfolded
Geometry and Measurement

Solid Figures

Sphere
Think of a ball
All radii are equal

Cone
\[ V = \frac{1}{3} \pi r^2 h \]
Its Volume is 1/3 of a cylinder with the same height and base

Pyramid
A square at the base with four triangles
\[ V = \frac{s^2 h}{3} \]

Problem 1: If the volume of a cube is 125 in\(^3\), what is the length of a side? What is the Surface Area of the cube?

Problem 2: If two cylinders have equal volume and the taller is four times higher than the shorter, what is the ratio of the radii?
Geometry and Measurement

**Coordinate Geometry**

Positive Slope  \[ y = 2x - 1 \]

Negative Slope  \[ y = -2x + 2 \]

Zero Slope  \[ y = 5 \]

Undefined Slope  \[ x = 4 \]

Two lines are **parallel** when their slopes are the same
\[ y = 2x + 3 \text{ is parallel to } y = 2x - 7 \text{ since the slope of both lines is 2} \]

Two lines are **perpendicular** when their slopes are negative reciprocals OR the product of the slopes is -1
\[ y = -2x + 3 \text{ is perpendicular to } y = \frac{1}{2}x - 7 \text{ since } (-2)(1/2) = -1 \text{ OR } (-2) \text{ is the negative reciprocal of } (1/2) \]

**Problem 1:** Give a line that is parallel to the line \( y = 3x - 4 \). Give a line that is perpendicular to it.

**Problem 2:** What is the distance of the two points (1,4) and (-1,-2)? What is their midpoint?

**Problem 3:** If (3,2) is the midpoint of two points, one being (-1,-2), what is the other point?

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**Midpoint Formula**
Given two points \((x_1, y_1)\) and \((x_2, y_2)\)
their midpoint is \((x_m, y_m)\)
where \(x_m = (x_1 + x_2)/2\) and \(y_m = (y_1 + y_2)/2\)

**Distance Formula**
Given two points \((x_1, y_1)\) and \((x_2, y_2)\)
their distance is \(d\)
where \(d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}\)
Geometry and Measurement

**Transformations**

- **Translation**
  - Moves up/down and left/right

- **Rotation**
  - Rotates on a point
  - Not necessarily the center
  - Clockwise

- **Reflection**
  - Reflects along a line of symmetry

**Problem 1:** If a clock is rotated 90 degrees clockwise, what number will be at the top?

**Problem 2:** If the triangle to the right is reflected about the y-axis, what are the new co-ordinates? If reflected about the x-axis, what are the new co-ordinates?

**Problem 3:** If the triangle to the right is translated 2 units up and 3 units left, what are the new co-ordinates? If then (after translation) it is reflected about the x-axis, what are the new co-ordinates?